Name Key

Date WK. first pg. 76-80 matching graphs

A *polynomial function* is of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

Where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers.

Leading coefficient: a_n

Degree: n

Constant: a_0

Common Polynomial Functions			
Degree	**Type**	**Standard Form**	**Example**
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1 x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2 x^2 + a_1 x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^4 + 2x - 1$

EXAMPLE 1 Identifying Polynomial Functions

Decide whether each function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a. $f(x) = -2x^3 + 5x + 8$ b. $g(x) = -0.8x^3 + \sqrt{2}x^4 - 12$

c. $h(x) = -x^2 + 7x^{-1} + 4x$ d. $k(x) = x^2 + 3^x$

a) yes - degree 3, cubic, LC = -2

b) yes - $g(x) = \sqrt{2}x^4 - .8x^3 - 12$, degree 4, quartic, LC = $\sqrt{2}$

c) no, $7x^{-1}$ → not a whole # → not a whole # exponent

d) no, 3^x → not a variable base

EXAMPLE 2 Evaluating a Polynomial Function

Evaluate $f(x) = 2x^4 - 8x^2 + 5x - 7$ when $x = 3$.

$f(3) = 2(3)^4 - 8(3)^2 + 5(3) - 7 = 98$

The **end behavior** of a function's graph is the behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$). For the graph of a polynomial function, the end behavior is determined by the function's degree and the sign of its leading coefficient.

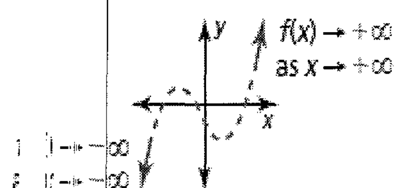
READING

The expression "$x \to +\infty$" is read as "x approaches positive infinity."

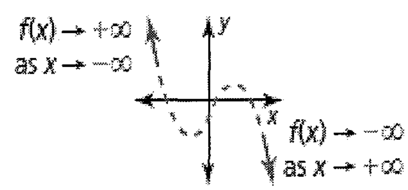
Core Concept

End Behavior of Polynomial Functions

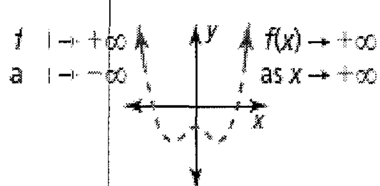
Degree: odd
Leading coefficient: positive



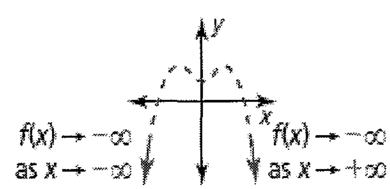
Degree: odd
Leading coefficient: negative



Degree: even
Leading coefficient: positive



Degree: even
Leading coefficient: negative



EXAMPLE 3 Describing End Behavior

Describe the end behavior of the graph of $f(x) = -0.5x^4 + 2.5x^2 + x - 1$.

Which 2 functions do we know the most about?
Linear - odd degree
Quad - even degree

degree 4
LC $(-)$ ↓↓

$f(x) \to -\infty$ as $x \to -\infty$ and
$f(x) \to -\infty$ as $x \to \infty$

Graphing Polynomial Functions

To graph a polynomial function, first plot points to determine the shape of the graph's middle portion. Then connect the points with a smooth continuous curve and use what you know about end behavior to sketch the graph.

EXAMPLE 4 Graphing Polynomial Functions

Graph (a) $f(x) = -x^3 + x^2 + 3x - 3$ and (b) $f(x) = x^4 - x^3 - 4x^2 + 4$.

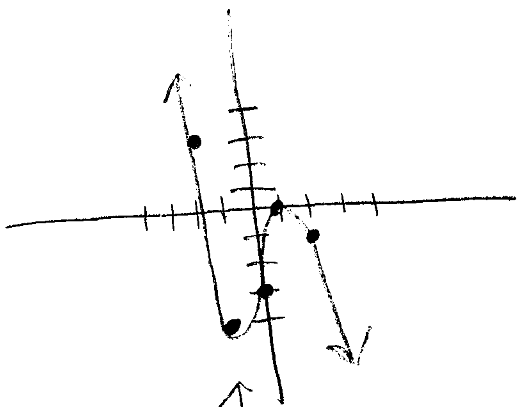
a)

x	y
−2	3
−1	−4
0	−3
1	0
2	−1

Odd

−

↑↓

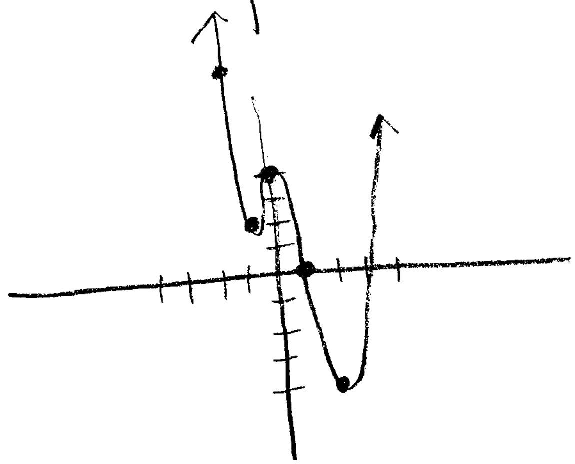


b)

x	y
−2	12
−1	2
0	4
1	0
2	−4

even

+

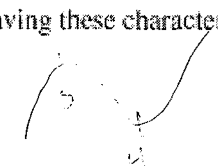


EXAMPLE 5 Sketching a Graph

Sketch a graph of the polynomial function f having these characteristics.

- f is increasing when $x < 0$ and $x > 4$.
- f is decreasing when $0 < x < 4$.
- $f(x) > 0$ when $-2 < x < 3$ and $x > 5$.
- $f(x) < 0$ when $x < -2$ and $3 < x < 5$.

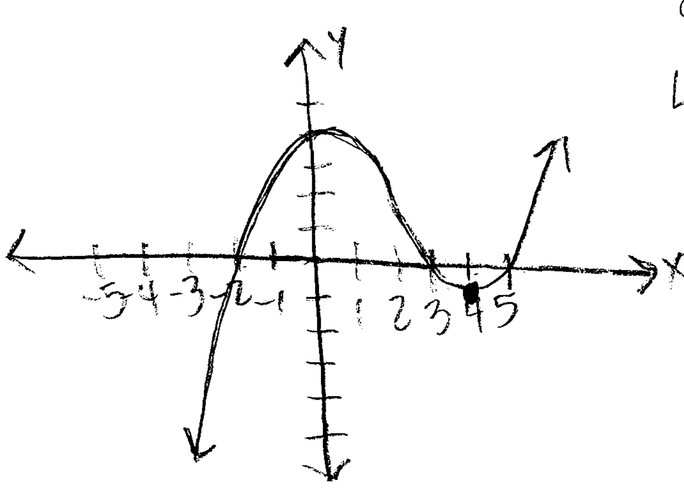
Use the graph to describe the degree and leading coefficient of f.

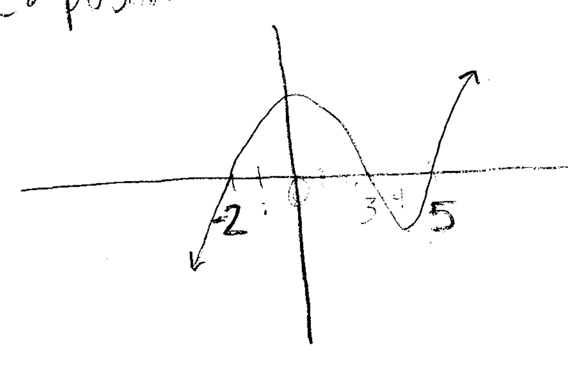


Do above 1st then breakdown between include

degree: Odd

LC: positive





EXAMPLE 6 Solving a Real-Life Problem

The estimated number V (in thousands) of electric vehicles in use in the United States can be modeled by the polynomial function

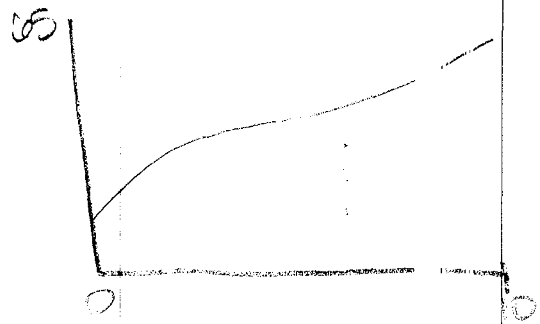
$$V(t) = 0.151280t^3 - 3.28234t^2 + 23.7[illegible]5t - 2.041$$

where t represents the year, with $t = 1$ corresponding to 2001.

a. Use a graphing calculator to graph the function for the interval $1 \le t \le 10$. Describe the behavior of the graph on this interval.

b. What was the average rate of change in the number of electric vehicles in use from 2001 to 2010?

c. Do you think this model can be used for years before 2001 or after 2010? Explain your reasoning.

a) 

From 2001 to 2004, the # of electric vehicles in use increased. Around 2005 the growth in #s in use slowed + started to level off. Then the #s in use started to inc again in 2009 + 2010.

b) Avg. rate of change: "slope"

2001 and 2010 correspond to $t=1$ and $t=10$

$$\frac{V(10)-V(1)}{10-1} = \frac{58.57-18.58444}{9} \approx 4.443$$

$\boxed{\text{4.4 thousand ele. veh. per yr.}}$

c) Degree: odd
LC: positive

Before 2001
$V(0) = -2.041 \rightarrow$ doesn't make sense (can't have neg. vehicles)

After 2010 $\rightarrow$ shows unlimited growth (which is not reasonable)